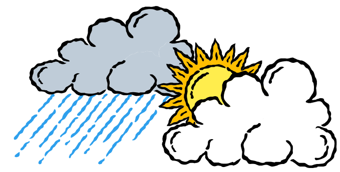
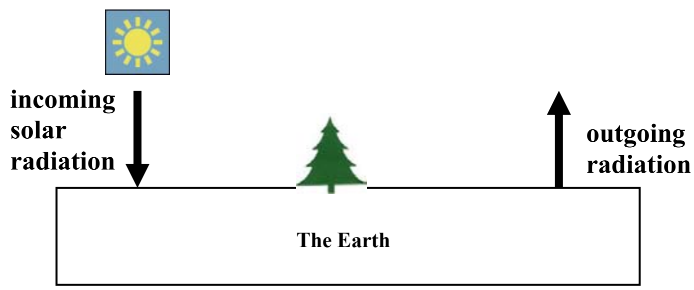
Climate



Data Sheet

The temperature of the Earth depends on the difference between the amount of **energy** the Earth receives from the Sun, and the amount of energy the Earth loses to space. For the Earth's average temperature to be steady, energy must leave at the same rate as it arrives. If energy arrives faster than it leaves, the Earth's temperature will rise.



The rate at which the Earth radiates energy back into space depends only on the temperature of the Earth, and is given by the Stefan-Boltzmann Law:

rate of outgoing radiation per square metre = $\boldsymbol{\sigma T^4}$

where $\boldsymbol{\sigma = 5.67 \times 10^{-8}\ \mathrm{Js^{-1}m^{-2}K^{-4}}}$ (the Stefan-Boltzmann constant)

The (average) temperature of the Earth's surface is 283 K, and so the energy lost per square metre of the Earth's surface per second is:

$$\text{Outgoing radiation} = \sigma T^4 = 364\ \mathrm{Js^{-1}m^{-2}} \quad \text{(to 3sf)}$$

The *change* in temperature of the Earth depends on the difference between the rate at which radiation arrives and the rate at which it leaves. It also depends on the heat capacity of the Earth (how easily the Earth heats up).

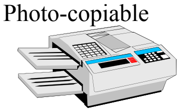
The heat capacity of the Earth = $\boldsymbol{4 \times 10^8\ \mathrm{JK^{-1}m^{-2}}}$; i.e. it takes 4×10^8 J to raise the temperature of 1 m^2 by 1 K.

The change in temperature of the Earth over one year is given by:

temperature change = (incoming radiation – outgoing radiation) × time / heat capacity

and the temperature of the Earth after one year will be given by:

new temperature = old temperature + temperature change



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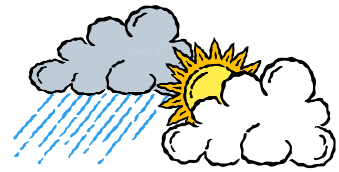

Worksheet

In this activity you will use a spreadsheet to predict what will happen if the incoming radiation changes from the equilibrium value of 364 $Js^{-1}m^{-2}$ when the temperature of the Earth is 283°K. Use the information and formulae on the Data Sheet.

- ***Open a new Excel spreadsheet.***

- ***First define the constants*** – put the value of σ in cell A1, the heat capacity of the Earth in cell B1 and a time increment in cell C1 – set this to half a year (making sure this is in seconds to be consistent with the other units).

- ***Put these table headings in cells A3 to E3:***
  Time, Incoming, Outgoing, Temperature change, Temperature

- ***Specify the initial conditions***
  In row 4 enter the temperature of the Earth, at time 0, to be 283 K and both the incoming and outgoing radiation to be 364 $Js^{-1}m^{-2}$.

Increase in incoming radiation

Follow the steps below to use the spreadsheet to predict what would happen if the incoming radiation increased by 5% and then remained constant:

- ***First consider the time (column A)***.
  Write a formula in A5 to give the time in A4 (0) plus the time increment.
  The time will be displayed in seconds – a big and not very meaningful number, so change your formula (not the constant!) to display the time in years.
  Copy the formula in column A down to row 20. This should give you a series of times at half yearly intervals.

- ***Now consider the incoming radiation (column B)***
  Write a formula in B5 to calculate the incoming energy after an increase of 5%.
  Complete column B down to row 20, keeping the incoming radiation constant at this new value.

- ***Next consider the outgoing radiation at time = 0.5 years (cell C5)***
  Construct a formula in cell C5 to give the outgoing radiation based on the constant in cell A1 and the temperature of the Earth at time 0 in cell C4.

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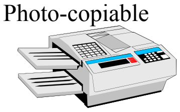


- ***Now consider the temperature change at time = 0.5 years (cell D5)***
  In D5, write a formula to calculate the temperature change:
  = (incoming radiation – outgoing radiation) × time increment / heat capacity

- ***Finally consider the temperature at time = 0.5 years (cell E5)***
  In E5 write a formula to calculate the new temperature:
  = old temperature + temperature change

- ***Extend the series in columns C, D and E down to row 20*** to see how temperature evolves in your model.

- ***Use the chart wizard to draw a graph of temperature against time*** for the Earth. Describe how the temperature changes with time.

Finding functions to model the way the temperature rises

- ***Make 4 copies of your graph.***

- ***Add a trendline as follows to the first 3 copies.***
  In each case use **Options** to set the intercept to 283 and display the equation.
  1st copy - add a ***quadratic*** regression line (polynomial order 2)
  2nd copy - add a ***cubic*** regression line (polynomial order 3)
  3rd copy - add a ***quartic*** regression line (polynomial order 4).

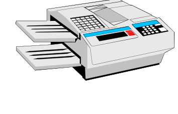
- ***Now try an exponential function:***
  Add another column to the spreadsheet to calculate values of the function
  $y = 283 + 3.53\left(1 - e^{-0.475t}\right)$ where t is time from column A.
  Add the graph of this function to the 4th copy of your graph.

Evaluating the functions as models of the temperature

Consider each of the functions you have found (quadratic, cubic, quartic and exponential) as models of this situation. For each model answer the following questions:

- How well does the model fit the temperature data?

- Do you think that the model gives realistic predictions for later times?
  (Note that you can quickly find out what the model predicts for later times by extending the columns further than 20 rows and then altering your graphs to include these new values.)



Decrease in incoming radiation

Now follow these steps to use the spreadsheet to predict what would happen if the incoming radiation decreased by 5% and then remained constant:

- ***Make a copy of the previous worksheet***

- ***Consider the incoming radiation (column B)***
  Change the formula in B5 so that it calculates the incoming energy after a decrease of 5%.

You should find that the other values in your spreadsheet change automatically (but if you set the scale on the *y* axis earlier you may need to change it now to show what is predicted to happen to the temperature).

- ***Describe what the graph of temperature against time shows now.***

Finding functions to model the way the temperature falls

- ***The new trendlines and their equations should be given on the first 3 graphs.***
  (If not, then redraw them.)

- ***Now alter the exponential function:***
  Change the formula in the last column of the spreadsheet so that it now calculates values of the function $y = 279.45 + 3.55e^{-0.45t}$
  where t is time from column A.

  Again Excel will update the graph (but you may need to alter the scale).

Evaluating the functions as models of the temperature

Consider each of the functions you have found (quadratic, cubic, quartic and exponential) as models of this situation. For each model answer the following questions:

- How well does the model fit the temperature data?

- Do you think that the model gives realistic predictions for later times?

Extension

If you have time, find models for other % increases and decreases.
You could also try using different time increments.

