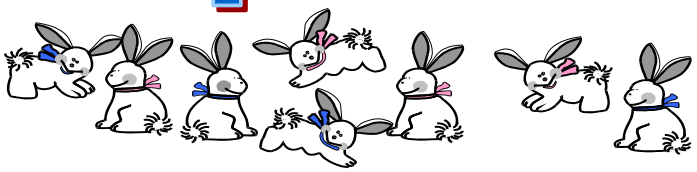
Chaotic Population



Teacher Notes

Unit Advanced Level, Applying mathematics

Notes
This is an adaptation of an activity that was written by Jon Gray (Banbury School) for a Nuffield project linked to climate*prediction*.net. Some information about the climate*prediction*.net project is given at the end of these notes.

This activity assumes that students have access to graphic calculators and Excel. It shows how to use a recurrence relation to simulate population changes. Some values of the parameter, k, lead to chaotic behaviour.

Answers The completed table and graph for $k = 2.4$ and $P_0 = 0.2$ are given below.

t	P_n
0	0.200
1	0.384
2	0.568
3	0.589
4	0.581
5	0.584
6	0.583
7	0.583
8	0.583
9	0.583
10	0.583
11	0.583
12	0.583
13	0.583
14	0.583
15	0.583

It can be seen that the population tends to a constant value of 0.583

Keeping $k = 2.4$ and using other values of P_0 give other sequences that behave in the same way.

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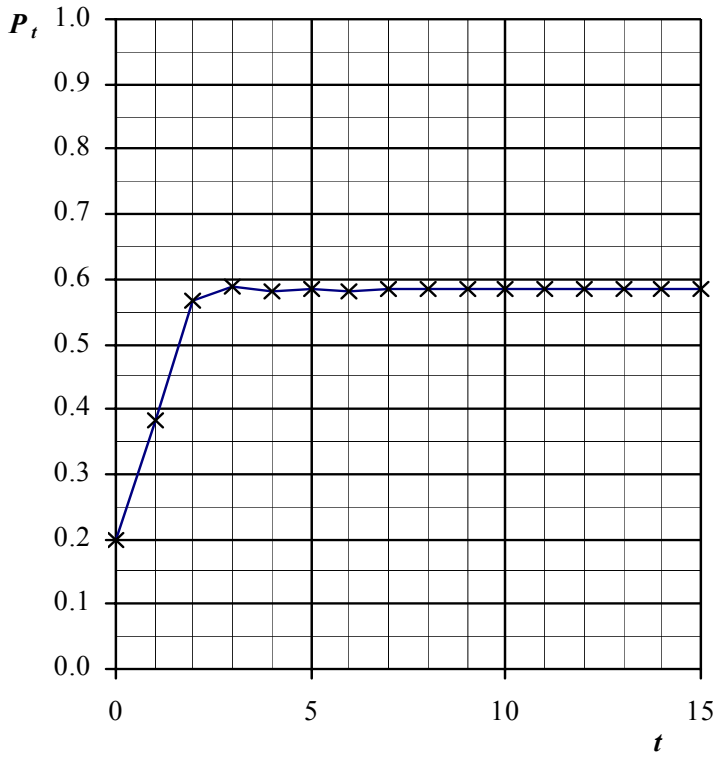
Population changes when $k = 2.4$ and $P_0 = 0.2$



For other values of k suggested on the spreadsheet, the population oscillates between increasing numbers of values until it starts to fluctuate chaotically at about 3.5 – a small change in the parameter, k, leading to large changes in the results. For values of 3.6 and above the behaviour of the population appears to be random, although there is some underlying structure. Students who are interested in investigating further could try plotting the $(n + 1)$th iteration against the nth. This gives an ***attractor***.

Similar situations arise in other real contexts such as weather. Although this activity is set in a different context, you may like to encourage students to find out more about climate by visiting the climate*prediction*.net website at www.climateprediction.net. Further activities linked to the project and information for teachers are also available from www.climateprediction.net/schools .

Climate*prediction*.net

This research project is a joint research project funded by the Natural Environment Research Council (NERC) and the Department of Trade and Industry. Its aim is to use the large number of idle computers worldwide and the power of the internet to predict and understand the climate. If they visit the website at www.climateprediction.net, your students can take part in this project by downloading their own unique simulation model of the Earth's climate. The downloaded program runs as a background process (it does not affect normal computing) to generate data for a climate model. The graphics packages supplied with the model show how weather patterns develop. Results from these experiments will contribute to the 4th Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) and will help policy makers plan for the effects of climate change.

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